**Integrals of simple functions**

*C* is used for an [arbitrary constant of integration](http://en.wikipedia.org/wiki/Arbitrary_constant_of_integration) that can only be determined if something about the value of the integral at some point is known. Thus each function has an infinite number of antiderivatives.

These formulas only state in another form the assertions in the [table of derivatives](http://en.wikipedia.org/wiki/Table_of_derivatives).

**Integrals with a singularity**

When there is a singularity in the function being integrated such that the integral becomes undefined, i.e., it is not [Lebesgue integrable](http://en.wikipedia.org/wiki/Lebesgue_integration), then *C* does not need to be the same on both sides of the singularity. The forms below normally assume the [Cauchy principal value](http://en.wikipedia.org/wiki/Cauchy_principal_value) around a singularity in the value of *C* but this is not in general necessary. For instance in

\int {1 \over x}\,dx = \ln \left|x \right| + C

There is a singularity at 0 and the integral becomes infinite there. If the integral above was used to give a definite integral between -1 and 1 the answer would be 0. This however is only the value assuming the Cauchy principal value for the integral around the singularity. If the integration was done in the complex plane the result would depend on the path around the origin, in this case the singularity contributes −*iπ* when using a path above the origin and *iπ* for a path below the origin. A function on the real line could use a completely different value of *C* on either side of the origin as in:

 \int {1 \over x}\,dx = \ln|x| + \begin{cases} A & \text{if }x>0; \\ B & \text{if }x < 0. \end{cases}  

**Rational functions**

*more integrals:* [*List of integrals of rational functions*](http://en.wikipedia.org/wiki/List_of_integrals_of_rational_functions)

These rational functions have a non-integrable singularity at 0 for *a* ≤ −1.

\int k\,dx = kx + C

\int x^a\,dx = \frac{x^{a+1}}{a+1} + C([Cavalieri's quadrature formula](http://en.wikipedia.org/wiki/Cavalieri%27s_quadrature_formula))

\int (ax + b)^n dx= \frac{(ax + b)^{n+1}}{a(n + 1)} + C \qquad\mbox{(for } n\neq -1\mbox{)}\,\!

\int {1 \over x}\,dx = \ln \left|x \right| + C

\int\frac{c}{ax + b} dx= \frac{c}{a}\ln\left|ax + b\right| + C

**Exponential functions**

*more integrals:* [*List of integrals of exponential functions*](http://en.wikipedia.org/wiki/List_of_integrals_of_exponential_functions)

\int e^x\,dx = e^x + C

\int a^x\,dx = \frac{a^x}{\ln a} + C

**Logarithms**

*more integrals:* [*List of integrals of logarithmic functions*](http://en.wikipedia.org/wiki/List_of_integrals_of_logarithmic_functions)

\int \ln x\,dx = x \ln x - x + C

\int \log_a x\,dx = x\log_a x - \frac{x}{\ln a} + C

**Trigonometric functions**

*more integrals:* [*List of integrals of trigonometric functions*](http://en.wikipedia.org/wiki/List_of_integrals_of_trigonometric_functions)

\int \sin{x}\, dx = -\cos{x} + C

\int \cos{x}\, dx = \sin{x} + C

\int \tan{x} \, dx = -\ln{\left| \cos {x} \right|} + C = \ln{\left| \sec{x} \right|} + C

\int \cot{x} \, dx = \ln{\left| \sin{x} \right|} + C

\int \sec{x} \, dx = \ln{\left| \sec{x} + \tan{x}\right|} + C

\int \csc{x} \, dx = -\ln{\left| \csc{x} + \cot{x}\right|} + C

\int \sec^2 x \, dx = \tan x + C

\int \csc^2 x \, dx = -\cot x + C

\int \sec{x} \, \tan{x} \, dx = \sec{x} + C

\int \csc{x} \, \cot{x} \, dx = -\csc{x} + C

\int \sin^2 x \, dx = \frac{1}{2}\left(x - \frac{\sin 2x}{2} \right) + C = \frac{1}{2}(x - \sin x\cos x ) + C 

\int \cos^2 x \, dx = \frac{1}{2}\left(x + \frac{\sin 2x}{2} \right) + C = \frac{1}{2}(x + \sin x\cos x ) + C 

\int \sec^3 x \, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C

(see [integral of secant cubed](http://en.wikipedia.org/wiki/Integral_of_secant_cubed))

\int \sin^n x \, dx = - \frac{\sin^{n-1} {x} \cos {x}}{n} + \frac{n-1}{n} \int \sin^{n-2}{x} \, dx

\int \cos^n x \, dx = \frac{\cos^{n-1} {x} \sin {x}}{n} + \frac{n-1}{n} \int \cos^{n-2}{x} \, dx

**Inverse trigonometric functions**

*more integrals:* [*List of integrals of inverse trigonometric functions*](http://en.wikipedia.org/wiki/List_of_integrals_of_inverse_trigonometric_functions)

\int \arcsin{x} \, dx = x \, \arcsin{x} + \sqrt{1 - x^2} + C

\int \arccos{x} \, dx = x \, \arccos{x} - \sqrt{1 - x^2} + C

\int \arctan{x} \, dx = x \, \arctan{x} - \frac{1}{2} \ln{\left| 1 + x^2\right|} + C

\int \arccot{x} \, dx = x \, \arccot{x} + \frac{1}{2} \ln{\left| 1 + x^2\right|} + C

\int \arcsec{x} \, dx = x \, \arcsec{x} - \operatorname{artanh}\,\sqrt{1-\frac{1}{x^2}} + C

\int \arccsc{x} \, dx = x \, \arccsc{x} + \operatorname{artanh}\,\sqrt{1-\frac{1}{x^2}} + C

**Hyperbolic functions**

*more integrals:* [*List of integrals of hyperbolic functions*](http://en.wikipedia.org/wiki/List_of_integrals_of_hyperbolic_functions)

\int \sinh x \, dx = \cosh x + C

\int \cosh x \, dx = \sinh x + C

\int \tanh x \, dx = \ln| \cosh x | + C

\int \mbox{csch}\,x \, dx = \ln\left| \tanh {x \over2}\right| + C

\int \mbox{sech}\,x \, dx = \arcsin\,(\tanh x) + C

\int \coth x \, dx = \ln| \sinh x | + C

**Inverse hyperbolic functions**

*more integrals:* [*List of integrals of inverse hyperbolic functions*](http://en.wikipedia.org/wiki/List_of_integrals_of_inverse_hyperbolic_functions)

\int \operatorname{arsinh} \, x \, dx=
    x \, \operatorname{arsinh} \, x-\sqrt{x^2+1}+C

\int \operatorname{arcosh} \, x \, dx=
    x \, \operatorname{arcosh} \, x-\sqrt{x+1} \, \sqrt{x-1}+C

\int \operatorname{artanh} \, x \, dx=
    x \, \operatorname{artanh} \, x+\frac{\ln\left(1-x^2\right)}{2}+C

\int \operatorname{arcoth} \, x \, dx=
    x \, \operatorname{arcoth} \, x+\frac{\ln\left(1-x^2\right)}{2}+C

\int \operatorname{arsech} \, x \, dx=
    x \, \operatorname{arsech} \, x-2 \, \arctan\sqrt{\frac{1-x}{1+x}}+C

\int \operatorname{arcsch} \, x \, dx=
    x \, \operatorname{arcsch} \, x+\operatorname{artanh}\sqrt{\frac{1}{x^2}+1}+C

**Products of functions proportional to their second derivatives**

\int \cos ax\, e^{bx}\, dx = \frac{e^{bx}}{a^2+b^2}\left( a\sin ax + b\cos ax \right) + C

\int \sin ax\, e^{bx}\, dx = \frac{e^{bx}}{a^2+b^2}\left( b\sin ax - a\cos ax \right) + C

\int \cos ax\, \cosh bx\, dx = \frac{1}{a^2+b^2}\left( a\sin ax\, \cosh bx+ b\cos ax\, \sinh bx \right) + C

\int \sin ax\, \cosh bx\, dx = \frac{1}{a^2+b^2}\left( b\sin ax\, \sinh bx- a\cos ax\, \cosh bx \right) + C

**Absolute value functions**

\int \left| (ax + b)^n \right|\,dx = {(ax + b)^{n+2} \over a(n+1) \left| ax + b \right|} + C \,\, [\,n\text{ is odd, and } n \neq -1\,]

\int \left| \sin{ax} \right|\,dx = {-1 \over a} \left| \sin{ax} \right| \cot{ax} + C

\int \left| \cos{ax} \right|\,dx = {1 \over a} \left| \cos{ax} \right| \tan{ax} + C

\int \left| \tan{ax} \right|\,dx = {\tan(ax)[-\ln\left|\cos{ax}\right|] \over a \left| \tan{ax} \right|} + C

\int \left| \csc{ax} \right|\,dx = {-\ln \left| \csc{ax} + \cot{ax} \right|\sin{ax} \over a \left| \sin{ax} \right|} + C 

\int \left| \sec{ax} \right|\,dx = {\ln \left| \sec{ax} + \tan{ax} \right| \cos{ax} \over a \left| \cos{ax} \right|} + C 

\int \left| \cot{ax} \right|\,dx = {\tan(ax)[\ln\left|\sin{ax}\right|] \over a \left| \tan{ax} \right|} + C 

**Special functions**

Ci, Si: [Trigonometric integrals](http://en.wikipedia.org/wiki/Trigonometric_integral), Ei: [Exponential integral](http://en.wikipedia.org/wiki/Exponential_integral), li: [Logarithmic integral function](http://en.wikipedia.org/wiki/Logarithmic_integral_function), erf: [Error function](http://en.wikipedia.org/wiki/Error_function)

\int \operatorname{Ci}(x) \, dx = x\,\operatorname{Ci}(x) - \sin x

\int \operatorname{Si}(x) \, dx = x\,\operatorname{Si}(x) + \cos x

\int \operatorname{Ei}(x) \, dx = x\,\operatorname{Ei}(x) - e^x

\int \operatorname{li}(x) \, dx = x\, \operatorname{li}(x)-\operatorname{Ei}(2 \ln x) 

\int \frac{\operatorname{li}(x)}{x}\,dx = \ln x\, \operatorname{li}(x) -x 

\int \operatorname{erf}(x)\, dx = \frac{e^{-x^2}}{\sqrt{\pi }}+x\, \text{erf}(x)

**Definite integrals lacking closed-form antiderivatives**

There are some functions whose antiderivatives *cannot* be expressed in [closed form](http://en.wikipedia.org/wiki/Closed-form_expression). However, the values of the definite integrals of some of these functions over some common intervals can be calculated. A few useful integrals are given below.

\int_0^\infty{\sqrt{x}\,e^{-x}\,dx} = \frac{1}{2}\sqrt \pi(see also [Gamma function](http://en.wikipedia.org/wiki/Gamma_function))

\int_0^\infty{e^{-a x^2}\,dx} = \frac{1}{2} \sqrt \frac {\pi} {a} (the [Gaussian integral](http://en.wikipedia.org/wiki/Gaussian_integral))

\int_0^\infty{x^2 e^{-a x^2}\,dx} = \frac{1}{4} \sqrt \frac {\pi} {a^3} when a > 0

\int_0^\infty{x^{2n} e^{-a x^2}\,dx}
= \frac{2n-1}{2a} \int_0^\infty{x^{2(n-1)} e^{-a x^2}\,dx}
= \frac{(2n-1)!!}{2^{n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}
= \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}}
when a > 0, n is 1,2,3,... and !! is the [double factorial](http://en.wikipedia.org/wiki/Double_factorial).

\int_0^\infty{x^3 e^{-a x^2}\,dx} = \frac{1}{2 a^2} when a > 0

\int_0^\infty{x^{2n+1} e^{-a x^2}\,dx}
= \frac {n} {a} \int_0^\infty{x^{2n-1} e^{-a x^2}\,dx}
= \frac{n!}{2 a^{n+1}}
when a > 0, n is 0, 1, 2, ....

\int_0^\infty{\frac{x}{e^x-1}\,dx} = \frac{\pi^2}{6}(see also [Bernoulli number](http://en.wikipedia.org/wiki/Bernoulli_number))

\int_0^\infty{\frac{x^2}{e^x-1}\,dx} = 2\zeta(3) \simeq 2.40

\int_0^\infty{\frac{x^3}{e^x-1}\,dx} = \frac{\pi^4}{15}

\int_0^\infty\frac{\sin{x}}{x}\,dx=\frac{\pi}{2}(see [sinc function](http://en.wikipedia.org/wiki/Sinc_function) and [Sine integral](http://en.wikipedia.org/wiki/Sine_integral))

\int_0^\infty\frac{\sin^2{x}}{x^2}\,dx=\frac{\pi}{2}

\int_0^\frac{\pi}{2}\sin^n{x}\,dx=\int_0^\frac{\pi}{2}\cos^n{x}\,dx=\frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (n-1)}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot n}\frac{\pi}{2}(if *n* is an even integer and   \scriptstyle{n \ge 2})

\int_0^\frac{\pi}{2}\sin^n{x}\,dx=\int_0^\frac{\pi}{2}\cos^n{x}\,dx=\frac{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (n-1)}{3 \cdot 5 \cdot 7 \cdot \cdots \cdot n}(if  \scriptstyle{n} is an odd integer and   \scriptstyle{n \ge 3} )

\int_{-\pi}^{\pi} \cos(\alpha x)\cos^n(\beta x) dx = \left \{ \begin{array}{cc}
\frac{2 \pi}{2^n} \binom{n}{m} & |\alpha|= |\beta (2m-n)| \\
0 & \mbox{otherwise} \\
\end{array} \right .(for \scriptstyle \alpha, \beta, m, nintegers with \scriptstyle \beta \neq 0and \scriptstyle m, n \geq 0, see also [Binomial coefficient](http://en.wikipedia.org/wiki/Binomial_coefficient))

\int_{-\pi}^{\pi} \sin(\alpha x) \cos^n(\beta x) dx = 0(for \scriptstyle \alpha,\betareal and \scriptstyle nnon-negative integer, see also [Symmetry](http://en.wikipedia.org/wiki/Symmetry))

\int_{-\pi}^{\pi} \sin(\alpha x) \sin^n(\beta x) dx = \left \{ \begin{array}{cc}
(-1)^{(n+1)/2} (-1)^m \frac{2 \pi}{2^n} \binom{n}{m} & n \mbox{ odd},\ \alpha = \beta (2m-n) \\
0 & \mbox{otherwise} \\
\end{array} \right .(for \scriptstyle \alpha, \beta, m, nintegers with \scriptstyle \beta \neq 0and \scriptstyle m, n \geq 0, see also [Binomial coefficient](http://en.wikipedia.org/wiki/Binomial_coefficient))

\int_{-\pi}^{\pi} \cos(\alpha x) \sin^n(\beta x) dx = \left \{ \begin{array}{cc}
(-1)^{n/2} (-1)^m \frac{2 \pi}{2^n} \binom{n}{m} & n \mbox{ even},\ |\alpha| = |\beta (2m-n)| \\
0 & \mbox{otherwise} \\
\end{array} \right .(for \scriptstyle \alpha, \beta, m, nintegers with \scriptstyle \beta \neq 0and \scriptstyle m,n \geq 0, see also [Binomial coefficient](http://en.wikipedia.org/wiki/Binomial_coefficient))

\int_{-\infty}^\infty e^{-(ax^2+bx+c)}\,dx=\sqrt{\frac{\pi}{a}}\exp\left[\frac{b^2-4ac}{4a}\right](where exp[*u*] is the [exponential function](http://en.wikipedia.org/wiki/Exponential_function) *eu*, and *a* > 0)

\int_0^\infty  x^{z-1}\,e^{-x}\,dx = \Gamma(z)(where Γ(*z*) is the [Gamma function](http://en.wikipedia.org/wiki/Gamma_function))

\int_0^1 x^{m-1}(1-x)^{n-1} dx = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} (the [Beta Function](http://en.wikipedia.org/wiki/Beta_Function))

\int_{0}^{2 \pi} e^{x \cos \theta} d \theta = 2 \pi I_{0}(x)(where *I*0(*x*) is the modified [Bessel function](http://en.wikipedia.org/wiki/Bessel_function) of the first kind)

\int_{0}^{2 \pi} e^{x \cos \theta + y \sin \theta} d \theta = 2 \pi I_{0} \left(\sqrt{x^2 + y^2}\right) 

\int_{-\infty}^{\infty}{(1 + x^2/\nu)^{-(\nu + 1)/2}dx} = \frac { \sqrt{\nu \pi} \ \Gamma(\nu/2)} {\Gamma((\nu + 1)/2)}\,, \nu > 0\,, this is related to the [probability density function](http://en.wikipedia.org/wiki/Probability_density_function) of the [Student's t-distribution](http://en.wikipedia.org/wiki/Student%27s_t-distribution))

The [method of exhaustion](http://en.wikipedia.org/wiki/Method_of_exhaustion) provides a formula for the general case when no antiderivative exists:

\int_a^b{f(x)\,dx} = (b - a) \sum\limits_{n = 1}^\infty  {\sum\limits_{m = 1}^{2^n  - 1} {\left( { - 1} \right)^{m + 1} } } 2^{ - n} f(a + m\left( {b - a} \right)2^{-n} ).

\int_0^1 [\ln(1/x)]^p\,dx = p!

**(Click "show" at right to see a proof or "hide" to hide it.)**[[show]](javascript:toggleNavigationBar(1);)

Start by using the substitution x = \operatorname{artanh}\,t

I_p = \int_0^1 [\ln(1/x)]^p\;\mathrm{d}x = \int^{\infty}_0 \left[\ln(1/\operatorname{artanh}\,t) \right]^p \;\frac{\mathrm{d}t}{1 - t^2}

This brings the integral to the general form

I_n = \int^b_a (\ln f)^n f^'\;\mathrm{d}t 

which after integration by parts yields

\left[f (\ln f)^n \right]^b_a - n \int^b_a (\ln f)^{n-1} f^'\;\mathrm{d}t 

and provided the first term vanishes at the end points, we get the recurrence relation

I_n = -n\,I_{n-1} 

which upon computation gives

I_n = (-1)^n\, n! 

Applying to our integral, we notice that

[\ln(1/x)]^p = (-1)^p\;[\ln(x)]^p 

Hence the final answer is:

I_p = (-1)^p\, (-1)^p\, p! = p!

**The "**[**sophomore's dream**](http://en.wikipedia.org/wiki/Sophomore%27s_dream)**"**

\begin{align}
\int_0^1 x^{-x}\,dx &= \sum_{n=1}^\infty n^{-n}        &&(= 1.29128599706266\dots)\\
\int_0^1 x^x   \,dx &= -\sum_{n=1}^\infty (-1)^n n^{-n} &&(= 0.783430510712\dots)
\end{align}

attributed to [Johann Bernoulli](http://en.wikipedia.org/wiki/Johann_Bernoulli).